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Train Localization using Unscented Kalman Filter – Based Sensor Fusion

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Abstract

This paper presents an application of sensor fusion methods based on Unscented Kalman filter (UKF) technique for solving train localization problem in rail systems. The paper first reports the development of a laboratory-scale rail system simulator which is equipped with various onboard and wayside sensors that are used to detect and locate the train vehicle movements in the rail track. Due to the low precision measurement data obtained by each individual sensor, a sensor fusion method based on the UKF technique is implemented to fuse the measurement data from several sensors. Experimental results which demonstrate the effectiveness of the proposed UKF-based sensor fusion method for solving the train localization problem is also reported.

Keywords

Sensor fusion; Train localization; Unscented kalman filter

1 Introduction

The increase in human population and needs lead indirectly to the need for transportation, especially in densely populated urban areas. Rail transportation system has been considered as one of the major potential solutions to modern land transportation problems. Factors such as higher speed, shorter travel time, high operational flexibility with lower cost/energy consumption are some of the advantages offered by railway systems over other means of land transportation [1,2].

Currently, the demand for train transportation services continues to increase and stimulates active research and rapid development of various technologies that can improve the operation of rail systems [3]. In particular, many studies and research activities have currently been done to find methods for increasing the carrying capacity, reducing the maintenance costs, as well as ensuring the high reliability and safety of the rail systems (cf. e.g. [3–5] and the references therein). More recently, advanced automation technologies have also been applied in railway systems through the introduction and use of Automatic Train Control (ATC) system. One significant feature of the ATC system is that the train operation is now mainly controlled by the computer instead of human [5]. Specifically, an embedded computer system on train (called onboard unit) is used to control the acceleration/deceleration of the train vehicle as it moves on the rail track. Clearly, the ability to use such a computerized system to automatically decide and execute movement commands heavily relies on the various position and velocity data that are measured using sensors installed on the train. It is thus clear that highly accurate measurement data from sensors are critically needed to achieve high precision control and assure reliable and safe operation of the rail system.

Due to the requirement of safety-critical operation, the use of high precision position and velocity sensors is considered to be a critical need in railway system [2]. Some of the commonly used sensors in rail systems include the transponder, axle counter, Inertial Navigation System (INS) and Global Positioning System (GPS) [2–3]. It is, however, important to realize that each of these sensors has its own advantages and weaknesses and so using their individual utilization may result in unsatisfactory results. One potential remedy for the limitation of relying only on a sensor measurement is the use of sensor fusion method [6–13]. In essence, sensor fusion is a method for combining sensory data or data derived from sensory data such that the resulting information is better (in a suitable sense) than what would possibly be achieved when these sources were used individually [6]. By using the sensor fusion method, the measured position and velocity data from sensors are expected to be more accurate.

Sensor fusion methods have been widely used in applications which involve automation technologies such as monitoring, target tracking, surveillance or robotics. An early application of sensor fusion in rail

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systems was introduced by Mirabadi et al. [7] wherein a fusion scheme for several measurements was used for train position and velocity estimation. In a more recent study, experimental result when using sensor fusion for railway localization was reported in [8].

This paper discusses some results of our experimental study in using sensor fusion method based on Unscented Kalman Filter (UKF) technique [6] to solve the train localization problem. In our experiment, a miniature train was developed and equipped with several sensors that are used as the train localization module. The used sensors include an Indoor Positioning System (IPS) for global position sensor, Inertial Measurement Unit (IMU) [14] to measure orientation of the train, and Radio Frequency Identification (RFID) sensor for train detection system. The data from these sensors are then fused using a UKF algorithm that was proposed by Freillard et al. in [9] to give better position and velocity estimates. Simulation results based on real experimental data are shown to illustrate the effectiveness of the proposed fusion method.

The remainder of this paper is structured as follows. Section 2 reviews and formulates the stochastic estimation problem in the context of sensor fusion techniques. Section 3 presents the concepts and algorithm of UKF. Section 4 describes the experimental setup used to implement the UKF-based sensor fusion method for solving the train localization problem. Section 5 gives the experimental results. Section 6 concludes the paper with remarks and discussion.

2 Preliminaries

This section briefly reviews the notion of stochastic estimation in Bayesian inference framework and formulates this estimation problem within the context of sensor fusion methods.

2.1 Stochastic estimation

Consider a dynamical system which evolves according to the following nonlinear discrete-time differential equation:

$$x_{k+1} = f(x_k, u_k, v_k), \qquad x(0) = x_0, \tag{1a}$$

$$y_k = h(x_k, n_k) \tag{1a}$$

In (1), x_k, u_k and y_k denote, respectively, the states, input and output/measurement at discrete time $k = 0,1,2,\cdots$ Variables v_k and n_k denote process and measurement noises, respectively, whereas both $f(\cdot)$ and $h(\cdot)$ are known functions. Our objective in this paper is to solve the stochastic estimation problem of (1). Specifically, given noisy measurement y_k of the system, our goal is to characterize the statistics (especially the first two moments) of the state variables vector x_k.

One solution to the above estimation problem is defined by the optimal estimate $\hat{x}_k$ of x_k which minimizes the mean-squared estimation error. This estimate is known as the Minimum Mean-Squared Error (MMSE) estimate and is formally defined as:

$$\hat{x}_k = E[x_k | Y_k] \tag{2}$$

with $E[\cdot|\cdot]$ denotes the conditional expectation and Y_k is the available measurements up to time k. The estimate in (2) can be obtained from the aposterior distribution of x_k which, by Bayes' rule, can be determined by the following Bayesian recursion [6]:

$$\mathrm{P}(x_k|Y_k) = \frac{\mathrm{P}(x_k|Y_{k-1})\mathrm{P}(y_k|x_k)}{\mathrm{P}(y_k|Y_{k-1})} \tag{3a}$$

with $\mathrm{P}(\cdot\,|\,\cdot)$ denotes the conditional probability while $\mathrm{P}(x_k|Y_{k-1})$ and $\mathrm{P}(y_k|Y_{k-1})$ satisfy:

$$P(x_k|Y_{k-1}) \;= \int P(x_k|x_{k-1})P(x_{k-1}|Y_{k-1})\,dx_{k-1} \tag{3b}$$

$$P(y_k|Y_{k-1}) = \int P(x_k|Y_{k-1})P(y_k|x_k) \tag{3c}$$

and $\mathrm{P}(x_k|x_{k-1})$ and $\mathrm{P}(y_k|x_k)$, respectively, are determined by the model in (1a) and (1b).

For arbitrarily distributed noises v_k and n_k in (1), the analytical solution of (3) is generally not available and so the recursion in (3) is usually solved using Monte Carlo type simulations. However, if these noises satisfy the Gaussian distribution, then (3) can be simplified into the following recursive computation of $\hat{x}_k$ and its covariance (C_{x_k}) [6]:

$$\hat{x}_k = \hat{x}_k^- + \mathbf{K}_k(y_k - \hat{y}_k^-) \tag{4a}$$

$$C_{x_k} = C_{x_k}^- + \mathbf{K}_k C_{\hat{y}_k} \mathbf{K}_k^T \tag{4b}$$

where $\hat{x}_k^-$ and $\hat{y}_k^-$ are the prediction of both x_k and y_k, respectively, while $\mathbf{K}_k$ is a gain factor. In particular, the optimal values (in MMSE sense) of $\hat{x}_k^-$, $\hat{y}_k^-$ and $\mathbf{K}_k$ satisfy [6]:

$$\hat{x}_k^- = E[f(x_{k-1}, u_{k-1}, v_{k-1})] \tag{5a}$$

$$\hat{y}_k^- = E[h(x_k, n_k)] \tag{5b}$$

$$\mathbf{K}_k = C_{x_k y_k} C^{-1}_{\tilde{y}_k \tilde{y}_k} \tag{5c}$$

in which: $\tilde{y}_k = y_k - \hat{y}_k^-$.

2.2 Problem formulation

Note in the Bayesian iteration (4) – (5) that one source of complexity is the computation of the expectations (5a) – (5b). When the system and output models in (1) are linear, the optimal MMSE estimate (2) is given by Kalman filter algorithm. When model (1) is nonlinear, the EKF may be used to its linearized, first order approximation. The use of low order approximation in EKF may, however, results in suboptimal or even divergent estimates. The UKF offers a remedy to this issue by proposing the use of a so-called Unscented Transformation (UT) to compute the estimate in (2). This paper is interested in formulating the UKF method for solving the estimation problem in a discrete-time dynamical system by using measurement data from several sensors. In particular, this paper examines the use of UKF-based sensor fusion method to estimate the position and velocity of train vehicle using several onboard and wayside sensor data.

3 Unscented Kalman Filter (UKF)

This section reviews the underlying principle and algorithmic implementation of UKF. The UKF method was originally developed to overcome issues that are often encountered in EKF. The basic idea in UKF method is the use of the so-called UT to form a set of minimum and carefully chosen sample points called sigma points which will then be used to estimate the distribution of a Gaussian random variable.

3.1 Unscented transformation

To understand the basic concept of UT, let x be an n-dimensional random variable with mean $\bar{x}$ and covariance P_x. Consider another random variable $y = f(x)$ which is obtained by propagating x through a nonlinear function $f(\cdot)$. Suppose our interest is in characterizing the statistics of y. The UT can be used for this purpose through the use of a set of sigma points that are constructed and stacked in a sigma matrix $\mathbf{S}^x$ with $(2n+1)$ sigma vectors $\sigma_i (i = 1, \cdots, n)$ below as it column elements:

$$\sigma_0 = \bar{x} \tag{6a}$$

$$\sigma_i = \bar{x} + \left(\sqrt{(n+\theta)P_x}\right)_i \tag{6b}$$

$$\sigma_{i+n} = \bar{x} - \left(\sqrt{(n+\theta)P_x}\right)_{i-n} \tag{6c}$$

in which $\theta = (\delta^2(n+\gamma) - n)$ denotes a scaling parameter, δ is a small constant specifying the distribution of the sigma points around $\bar{x}$ and $\gamma = 3 - n$ is also a scaling parameter.

The UKF approximates the statistics (i.e., first two moments) of y by computing the weighted sample mean and covariance of another set of random variables $\mathrm{Y_i}$ which results from propagating the sigma point σ_i through an appropriate function of the form:

$$\mathrm{Y_i} = f(\sigma_i), \qquad i = 1, \cdots, 2n \tag{7}$$

More specifically, the mean and covariance of y is approximated from the samples of $\mathrm{Y_i}$ as:

$$\bar{y} \approx \textstyle\sum_{i=0}^{2n} w_i^1 Y_i \tag{8a}$$

$$P_y \approx \textstyle\sum_{i=0}^{2n} w_i^2 (Y_i - \bar{y})(Y_i - \bar{y})^T \tag{8b}$$

where the weights w_i with $i = 1, \cdots, 2n$ for the first and second moments of y are defined as:

$$w_0^1 = \frac{\gamma}{n+\gamma} \tag{9a}$$

$$w_0^2 = 1 + w_0^1 + \delta^2 - \eta \tag{9b}$$

$$w_i^1 = w_i^2 = \frac{1}{2(n+\gamma)} \tag{9c}$$

with η is a parameter which captures prior information about the distribution of x (e.g., $\eta = 2$ if x has a Gaussian distribution) [6,13].

3.2 UKF algorithm

The UKF approach to computing the estimate (2) for the process in (1) basically applies the aforementioned UT on a vector of augmented state variables below:

$$z_k = [x_k \quad v_k \quad n_k]^T \tag{10}$$

Correspondingly, the sigma matrix Σ^z for this augmented state should also be generated by the UT with respect to each element of z_k in (10). Thus, this sigma matrix ($\mathbf{S}^z$) is defined as:

$$\mathbf{S}^z = [\mathbf{S}^x \quad \mathbf{S}^v \quad \mathbf{S}^n]^T \tag{11}$$

where each $\mathbf{S}^x$,$\mathbf{S}^v$ and $\mathbf{S}^n$ is constructed iteratively according to the iteration in (6). The UKF algorithm essentially consists of four main steps, namely the initialization, sigma point generation, prediction and correction or update steps. The routines in each of these steps are described below [6,13].

Step* 1: *Initialization

Initialize the augmented state estimate $\hat{z}_0$ and the corresponding covariance C_z^0 as:

$$\hat{z}_0 = \mathrm{E}[\hat{x}_0 \quad 0 \quad 0] \tag{12a}$$

Where: $\hat{x}_0 = E[x_0]$

$$C_z^0 = \mathrm{E}[(z_0 - \hat{z}_0)(z_0 - \hat{z}_0)^T] = \begin{bmatrix} C_x^0 & 0 & 0 \\ 0 & C_v & 0 \\ 0 & 0 & C_n \end{bmatrix} \tag{12b}$$

with $C_x^0 = \mathrm{E}[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$, while C_v and C_n are the covariance of v_k and n_k, respectively.

Step* 2: *Sigma Point Generation

For $k = 1, \cdots, \infty$, generate the sigma matrix of the augmented state as follows:

$$\mathbf{S}_{k-1}^z = \begin{bmatrix} z_{k-1} \\ z_{k-1} + (n+\theta)\sqrt{C_{k-1}^z} \\ z_{k-1} - (n+\theta)\sqrt{C_{k-1}^z} \end{bmatrix} \tag{13}$$

Step* 3: *Prediction

For $k = 1, \cdots, \infty$, propagate each element of the sigma matrix of z_k iteratively through model equation (1) to get the updated sigma matrix $\Sigma_{k|k-1}^x$ of the state and the corresponding measurement samples $Y_{k|k-1}$ below:

$$\mathbf{S}_{k|k-1}^x = f(\mathbf{S}_{k-1}^x, u_{k-1}, \mathbf{S}_{k-1}^v) \tag{14a}$$

$$Y_{k|k-1} = h(\mathbf{S}_{k|k-1}^x, \mathbf{S}_{k-1}^n) \tag{14b}$$

Use the above $\mathbf{S}_{k|k-1}^x$ and $Y_{k|k-1}$ to compute the prediction of the state estimate, covariance and output variables in the following iteration:

$$\hat{x}_k^- = \sum_{i=0}^{2n} w_i^1 \, \mathbf{S}_{i,k|k-1}^x \tag{15a}$$

$$C_k^- = \sum_{i=0}^{2n} w_i^2 \left(\mathbf{S}_{i,k|k-1}^x - \hat{x}_k^-\right)\left(\mathbf{S}_{i,k|k-1}^x - \hat{x}_k^-\right)^T \tag{15b}$$

$$\hat{y}_k^- = \sum_{i=0}^{2n} w_i^1 \, Y_{i,k|k-1}^x \tag{15c}$$

Step* 4: *Correction

Update the state and the covariance of the estimate with the following iteration:

$$\hat{x}_k = \hat{x}_k^- + \mathbf{K}_k(y_k - \hat{y}_k^-) \tag{16a}$$

$$C_k = C_k^- - \mathbf{K}_k C_{\widetilde{y_k}\widetilde{y_k},} \mathbf{K}_k^T \tag{16b}$$

Where:

$$K_k = C_{x_k y_k} C_{\widetilde{y_k}\widetilde{y_k}}^{-1}$$

$$C_{x_k y_k} = \sum_{i=0}^{2n} w_i^2 \left(\mathbf{S}_{i,k|k-1} - \hat{x}_k^-\right)\left(\mathbf{S}_{i,k|k-1} - \hat{y}_k^-\right)^T$$

$$C_{\widetilde{y_k}\widetilde{y_k}} = \sum_{i=0}^{2n} w_i^2 \left(Y_{i,k|k-1} - \hat{y}_k^-\right)\left(Y_{i,k|k-1} - \hat{y}_k^-\right)^T$$

4 System Description and Setup

4.1 System architecture

Figure 1 shows the train miniature which was developed for our experiment and equipped with various instruments to mimic the actual functionality and operation of a real railway system. On the wayside of the miniature track, four IPS stationary beacons and six RFID tags are used simultaneously as the train localization system.

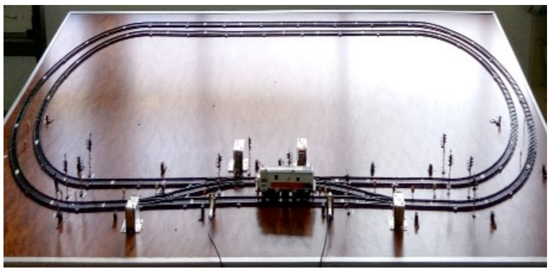

Figure 1 Experimental train miniatures

The stationary beacons play the role of small satellites which periodically transmits signal to the receiver; reminiscent to GPS functionality. The IPS used in our setup is the Marvelmind Indoor Navigation System (MINS) developed by Marvelmind Robotics [15]. The MINS' working principle is based on the propagation delay of ultrasonic waves and can provide precise location (±2cm accuracy) of an indoor object. The GPS was not used in the setup due to its low performance in locating indoor moving objects. Specifically, we found out that the GPS could not meet the miniature's accuracy requirement (which is 25 times smaller than what can be provided by the best GPS).

The RFID tags were located on several known positions on the track and play the role of axle counters which detect the presence of a train vehicle in a particular trail segment. Figure 2 shows the schematic of the miniature with the locations of stationary beacons and RFID tags.

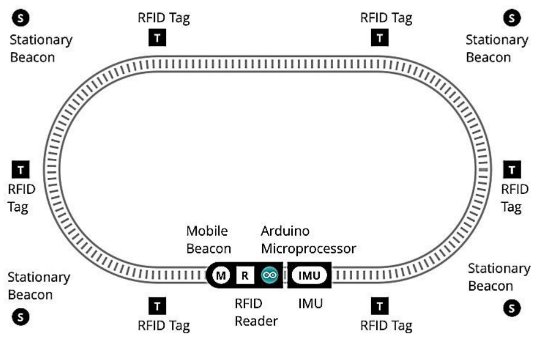

Figure 2 Schematic of the miniatures

The onboard unit of the miniature is equipped with an IMU containing gyroscope and accelerometer, a mobile IPS receiver beacon, an RFID reader and an Arduino microprocessor board. The mobile IPS beacon continuously receives signals from the stationary one containing information about its position in the two-dimensional Cartesian coordinate system. The RFID reader reads the RFID tags that it passes through and uses the tags position information to correct the train position. Both the gyro and accelerometer measure the train orientation with high sampling rates to provide aided positional information in the absence of RFID tags. All of these sensors information is collected and sent to the computer by the Arduino board to be fused and processed using the UKF algorithm.

4.2 Train kinematic model

The implementation of the UKF-based sensor fusion is conducted using the kinematic model of the miniature's train. For this purpose, a first-order differential equation model below describing the kinematics of the train is used:

$$p_k = p_{k-1} + v_k \cdot \Delta t \tag{17}$$

where p_k denotes the estimate of the train position at time k while p_{k-1} and v_k denote, respectively, the train position and velocity at time $k-1$. Expanding the kinematic (17) to two-dimensional Cartesian coordinate, one has:

$$\begin{bmatrix} p_k^x \\ p_k^y \end{bmatrix} = \mathbf{I}_{2\times2} \begin{bmatrix} p_{k-1}^x \\ p_{k-1}^y \end{bmatrix} + \Delta t \cdot \mathbf{I}_{2\times2} \begin{bmatrix} v_k^x \\ v_k^y \end{bmatrix} \tag{18}$$

where $\mathbf{I}_{2\times2}$ denotes a (2×2) identity matrix while the input velocities v_k^x and v_k^y in the x and y coordinates, respectively, satisfy:

$$v_k^x = v_k \cos\emptyset, \qquad v_k^y = v_k \sin\emptyset \tag{19}$$

with v_k denotes the train speed at time k and $\emptyset$ is the train orientation which in particular is defined as the following yaw value:

$$\emptyset = \int \omega_\phi \, dt \tag{20}$$

which can be computed from the measured angular velocity obtained by the gyro. Finally, it is assumed that the available measurements include the position in $x-y$ coordinate, i.e.:

$$y_k = \mathbf{I}_{2\times2} p_k \tag{21}$$

The UKF method was implanted to compute the estimation of the train position based on the position measurements $\widetilde{y_k}$ which has obtained from the RFIDs. The estimate was computed using an assumption that the variance of the RFID measurement is one-tenth of the variance of the gyroscope. The covariance matrices used in the UKF were therefore assumed to be $C_v = \mathbf{I}_{2\times2} \cdot var(v)$ and $C_n = \mathbf{I}_{2\times2} \cdot var(n) \cdot \Delta t$

5 Experimental Results

5.1 Individual sensor estimation

In the experiment, we first examine the results of position estimation that are obtained by each individual sensor.

For the IPS sensor experiment, one mobile beacon was installed on the moving train while four stationary beacons were installed at the four corners of the miniature's platform. As the train moves on the track, the MINS was used to estimate the train position and the obtained result is plotted against the actual train position in Figure 3. It can be seen in this figure that the IPS estimates closely aligns the actual track of the miniature.

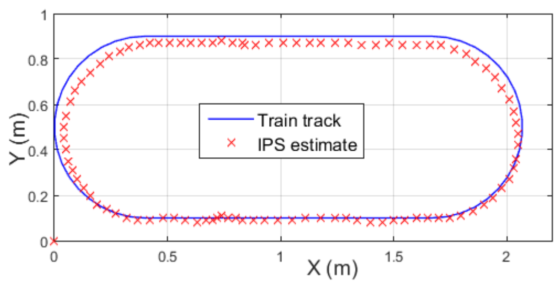

Figure 3 Estimation result of IPS sensor

In the second experiment, the IMU sensor is used to estimate the train position and velocity based on the measured train acceleration and yaw. Figure 4 shows one example of the measured yaw obtained from the IMU data. This data is then used to estimate the train velocity as shown in Figure 5.

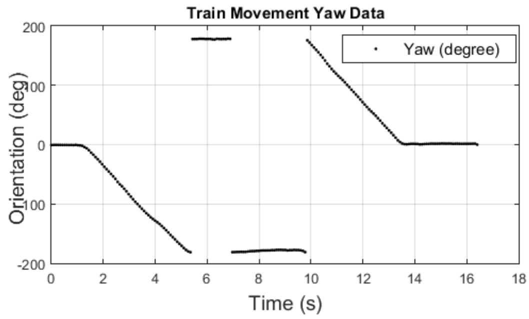

Figure 4 Measured yaw from IMU sensor

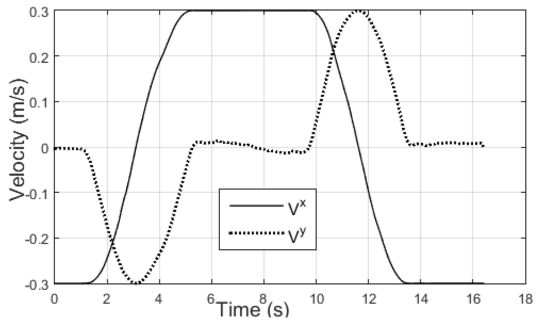

Figure 5 Train velocity estimate

The velocity estimate is then projected onto the train position with respect to the track. The obtained estimate of the train position based on the IMU sensor data is plotted in Figure 6. This plot shows that the resulting train position poorly estimates the actual train track.

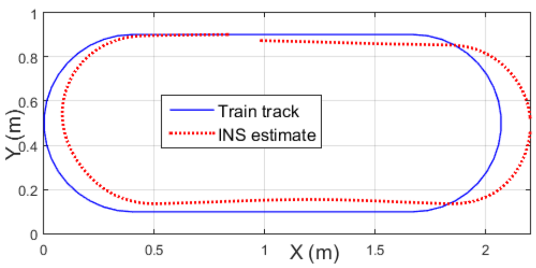

Figure 6 Estimation result of IMU sensor

Based on the results obtained from individual sensor experiment, it is clear that improvement is needed for the case of IMU-based position estimation. For this reason, another experiment was done to implement the sensor fusion algorithm with the goal of improving the result obtained from the IMU-based estimate.

5.2 Estimation using sensor fusion

In the third experiment, we implemented the UKF-based sensor method to estimate the train position based on the fusion of measurement data from IMU and RFID. In this regard, the RFID data is used to correct and improve the IMU estimate through the use of UKF method described in Section 3.

Figure 7 shows the obtained position estimate of the train when using the UKF-based sensor fusion method. It is clear from the result plotted in this figure that the proposed UKF-based sensor fusion method for estimating the train position outperforms the IMU-based one.

Performance comparison between different estimation methods that were examined in the experiments was also performed. For this purpose, the Root Mean Squared Error (RMSE) is used as the performance criterion. Table 1 lists the RMSE of each approach when used to estimate the train position. It can be seen that the UKF-based estimation method gives the smallest RMSE in both the x and y axis and thus outperforms the single sensor approach based on either the IPS or IMU sensor.

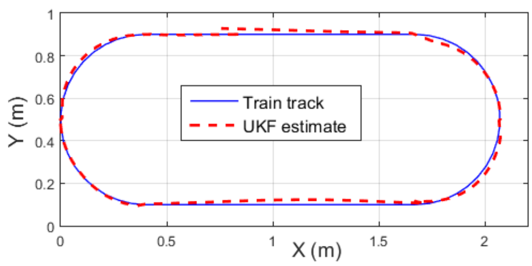

Figure 7 Estimation result obtained using UKF-based sensor fusion

Table 1 Summary of RMSE

Estimation Method	Axis	RMSE
UKF-based IMU + RFID	x	0.02897483
	y	0.02849436
IPS Sensor	x	0.06389007
	y	0.05212586
IMU Sensor	x	0.10260048
	y	0.03589154

6 Remark and Discussion

This paper has presented some results from an experimental application of UKF-based sensor fusion method when used to solve the train localization problem in railway system. Through the combination of measurement data from an IMU sensor and RFID system, it was shown that the results obtained using the proposed sensor fusion method improve those obtained using individual sensor measurement data. The proposed sensor fusion method also outperforms a global IPS sensor system in term of the resulting mean squared error.

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